The Fishery Model
of
Internationally Exploited Species*

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The purpose of this paper is to revise my original paper in this journal ("The Management of the Tuna Species: The Case of the Eastern Tropical Pacific," Vol. 13, No. 4, Dec. 1979). In the follwoing, I will discuss the revised fishery model of internationally exploited species (chapter 2).

CHAPTER 2 (revised)

THE FISHERY MODEL OF INTERNATIONALLY EXPLOITED SPECIES

The purpose of this chapter is to analyze the various policy measures under an open-access situation. First, we will examine the properties of the model of internationally exploited fishery resources. Then, the impact of the various policies will be examined.[1]

In the following, only the open-access situation is assumed to prevail in two countries. We assume that two countries, i.e., country X and Y, operate in one fishing ground. To investigate the effects of the restrictive measures such as the imposition of a landings tax and a licensings fee, two basic equations are shown as follows:

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$$(1-t_x)P_xF_x-(c_x+l_x)E_x=0 \tag{1'}$$

$$(1-t_y)P_yF_y-(c_y+l_y)E_y=0, \tag{2'}$$

where t_i $(i=x, y)$ is a landings tax rate, P_i is a price of tuna in each domestic market, c_i is a marginal cost, and l_i is a licensings fee.

F_i shows the amount of catches in each country as given by equation (3).[2]

$$F_i=(E_i/(E_x+E_y))(a(E_x+E_y)-b(E_x+E_y)^2), \tag{3}$$

where E_x and E_y are the amount of the fishing efforts of country X and Y, respectively.

Equations (1)′ and (2)′ show the open-access situations. $(1-t_i)\times P_iF_i$ corresponds to the total net revenue, and $(c_i+l_i)E_i$ is the total cost. Therefore, the net profit of the industry in country X and Y (i.e., Π_x and Π_y, respectively) becomes zero in the long-run equilibrium situation.

Demand for tuna is expressed as follows:[3]

$$P_i=\alpha_i-\gamma_iF_i \qquad (i=x, y). \tag{4}$$

Substituting equation (4) into equations (1)′ and (2)′, and setting $\alpha_i=\gamma_i=1$ by taking the appropriate units of measurements, a pair of simplified equations (1), and (2) are given by dividing (1)′ by E_x and (2)′ by E_y, respectively.

$$(1-E_x\Lambda)\Lambda-\Gamma_x=0 \tag{1}$$

$$(1-E_y\Lambda)\Lambda-\Gamma_y=0, \tag{2}$$

where $\Gamma_i\equiv(c_i+l_i)/(1-t_i)$ $(i=x, y)$ and $\Lambda\equiv a-b(E_x+E_y)$.

In order to examine the effects of the imposition of a landings tax and licensings fee, equations (1) and (2) must be totally differentiated. By taking the appropriate units of measurements, we can set the initial values $E_x=E_y=1$. Equations (1) and (2) can be differentiated totally as follows by noticing $\Lambda=a-2b$, and

$d\Lambda = -b(dE_x + dE_y)$:

$$(6ab - a^2 - 8b^2 - b)dE_x + (2ab - b - 4b^2)dE_y = d\Gamma_x \tag{3}$$

$$(2ab - b - 4b^2)dE_x + (-a^2 + 6ab - b - 8b^2)dE_y = d\Gamma_y. \tag{4}$$

To simplify the coefficients of equations (3) and (4), we suppose that the initial level of the world fishing efforts are on MSY level, i.e., $a/2b = E_x + E_y$. Recalling $E_x = E_y = 1$ initially, $a = 4b$ is obtained at the initial MSY level.

By substituting $a = 4b$ into equations (3) and (4), a pair of the simplified equations in matrix form is given as shown in equation (5).

$$\begin{pmatrix} -b & 4b^2 - b \\ 4b^2 - b & -b \end{pmatrix} \begin{pmatrix} dE_x \\ dE_y \end{pmatrix} = \begin{pmatrix} d\Gamma_x \\ d\Gamma_y \end{pmatrix}. \tag{5}$$

In order to determine the sign of the determinant of the lefthand side of the coefficients in equation (5), a stability condition of the model should be examined. We suppose that each country increases her fishing efforts as far as there exists a positive net profit, i.e., $\pi_i > 0$ $(i = x, y)$. Based upon this assumption, the adjustment process is shown as follows:

$$\begin{aligned} E_x &= \delta_1 \pi_x;\ \delta_1 > 0 \\ E_y &= \delta_2 \pi_y;\ \delta_2 > 0. \end{aligned} \tag{6}$$

By taking into account the initial values $t_i = l_i = 0$, and $E_i = 1$, the elements of the jacobian of equation (6) are given in equation (7) by differentiating equations (1)′ and (2)′ by E_x and E_y.

$$\begin{aligned} \partial\pi_x/\partial E_x &= -(a-3b)(a-2b) + (1-a+2b)(a-3b) - c_x \\ \partial\pi_x/\partial E_y &= b(a-2b) - b(1-a+2b) \\ \partial\pi_y/\partial E_x &= b(a-2b) - b(1-a+2b) \\ \partial\pi_y/\partial E_y &= -(a-3b)(a-2b) + (1-a+2b)(a-3b) - c_y. \end{aligned} \tag{7}$$

By setting $\pi_x = \pi_y = 0$ initially, $c_i = (a-2b)(1-(a-2b)) = 2b(1-2b)$ hold. Substituting $a = 4b$ into equation (7), a simplified jacobian matrix is obtained in equation (8).

$$J \equiv \begin{pmatrix} \partial\pi_x/\partial E_x & \partial\pi_x/\partial E_y \\ \partial\pi_y/\partial E_x & \partial\pi_y/\partial E_y \end{pmatrix}$$

$$= \begin{pmatrix} b-4b^2-2b(1-2b) & 4b^2-b \\ 4b^2-b & b-4b^2-2b(1-2b) \end{pmatrix}$$

$$= \begin{pmatrix} -b & 4b^2-b \\ 4b^2-b & -b \end{pmatrix}. \tag{8}$$

The necessary conditions for the stability are as follows:

$$\begin{aligned} &-b<0 \\ &\det J = b^2-(4b^2-b)^2 \\ &\quad = 8b^3(1-2b)>0. \end{aligned} \tag{9}$$

From equation (9), the necessary conditions obtained as $0<b<1/2$.

Let Δ be the determinant of the lefthand side coefficient matrix in equation (5), Δ is positive as far as $0<b<1/2$ holds.

$$\begin{aligned} \Delta &\equiv b^2-(4b^2-b)^2 \\ &= 8b^3(1-2b)>0 \text{ as } 0<b<1/2. \end{aligned}$$

Solving equation (5) for dE_x and dE_y by setting $d\Gamma_y=0$, equation (10) is obtained.

$$\begin{aligned} dE_x/d\Gamma_x &= -b/\Delta<0 \\ dE_y/d\Gamma_x &= -(1/\Delta)4b(b-1/4)>0 \text{ for } 0<b<1/4 \\ &\qquad <0 \text{ for } 1/4<b<1/2. \end{aligned} \tag{10}$$

The impact of changes in $d\Gamma_i = dc_i + dl_i + c_i dt_i$ is obtained by differentiating $\Gamma_i=(c_i+l_i)/(1-t_i)$ totally. In the following, the effects of the imposition of the landings tax in country X is examined. From equation (10), the results as shown below are obtained by setting $d\Gamma_i = c_x dt_x = 2b(1-2b)dt_x$.

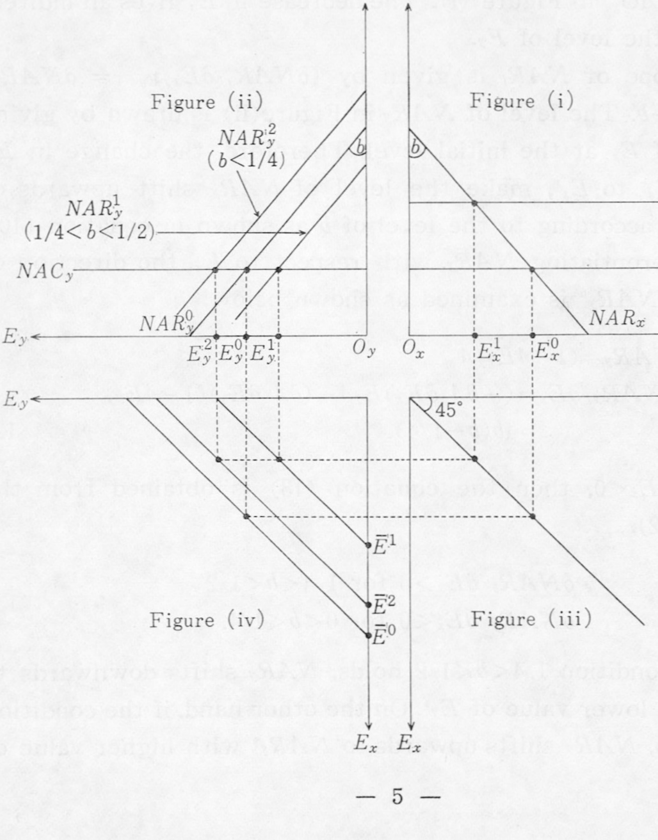
$$dE_x/dt_x = (-b/8b^3(1-2b))2b(1-2b)$$

$$=-1/4b$$

$$dE_y/dt_x=-(1/8b^3(1-2b))4b(b-1/4) \qquad (11)$$

$$=(b-1/4)/2b^2(b-1/2).$$

The increase in the tax rate in country X reduces the revenue of country X. This reduction causes the net revenue (π_x) to be negative from the initial situation $\pi_x=0$. From the stability condition, the level of efforts in country X, i.e., E_x, reduces to downwards, which corresponds to $dE_x/dt_x=-1/4b>0$ in equation (11). This direct impact on the levels of E_x leads to the indirect impact on the level of E_y. To make a clear explanation, the Figure (i)-(iv) are drawn as shown below.



In Figure (i), NAC_x shows Γ_x. Dividing average cost curve, i.e., $(c_x+l_x)E_x/E_x=c_x+l_x$, by $(1-t_x)$, Γ_x is obtained and named *normalized average cost curve* of country X, i.e., NAC_x.

NAR_x corresponds to $(1-E_x\Lambda)\Lambda$ in equation (1) with the same procedures as NAC_x. E_x^0 is therefore, the initial open-access equilibrium fishing efforts in the country X given the country Y's efforts level at E_y^0.

In Figure (ii), NAC_y and NAR_y are drawn by using the same procedure as done in Figure (i). For the convenience of explanation, the initial level of efforts are drawn as $O_xE_x^0=O_yE_y^0=1$.

Increase in tax rate in the country X is shown as a shift of NAC_x to NAC_x' in Figure (i). The decrease in E_x gives an indirect impact on the level of E_y.

The slope of NAR_i is given by $(\partial NAR_x/\partial E_x)_{E_y=1}=(\partial NAR_y/\partial E_y)_{E_x=1}=-b$. The level of NAR_y in Figure(ii) is drawn by giving the level of E_x at the initial level. Therefore, the change in E_x, i.e., from E_x^0 to E_x^1, make the level of NAR_y shift upwards or downwards according to the level of b as shown in equation (10).

By differentiating NAR_y with respect to E_x, the direction of shift from NAR_y^0 is examined as shown below:

$$NAR_y=(1-\Lambda E_y)\Lambda$$
$$\partial NAR_y/\partial E_x=(-\partial\Lambda/\partial E_x)E_y\Lambda+(\partial\Lambda/\partial E_x)(1-\Lambda E_y)$$
$$=4b(b-1/4). \tag{12}$$

Take $dE_x<0$, then the equation (13) is obtained from the equation (12).

$$\begin{aligned}&\partial NAR_y/\partial E_x>0 \text{ for } 1/4<b<1/2\\&\partial NAR_y/\partial E_x<0 \text{ for } 0<b<1/4.\end{aligned} \tag{13}$$

If the condition $1/4<b<1/2$ holds, NAR_y shifts downwards to NAR_y^1 with lower value of E_y^1. On the other hand, if the condition $b<1/4$ holds, NAR_y shifts upwards to NAR_y^2 with higher value of

E_y^2. This shifts corresponds to the results obtained in equation (10).

There accrues a net surplus loss in the country X, on the other hand, there accrues a net surplus gain in the country Y if $b<1/4$ holds. In this circumstances, the fishermen in the country X do not approve the unilateral imposition of a landings tax, and they also claim a bilateral imposition of a tax in the country Y. By setting $d\Gamma_x=d\Gamma_y=d\Gamma$, the effect of the bilateral tax imposition is obtained as shown in equation (14).

$$
\begin{aligned}
dE_x/d\Gamma &= -4b^2/\Delta<0 \\
dE_y/d\Gamma &= -4b^2/\Delta<0.
\end{aligned} \tag{14}
$$

Figure (iv) shows the total impact of the tax imposition in the country X on the world fishing efforts.[4] Total effects are obtained as shown in equation (15) and (16).

$$
\begin{aligned}
dE/d\Gamma_x &= dE_x/d\Gamma_x + dE_y/d\Gamma_x \\
&= -b/\Delta-(1/\Delta)(4b^2-b) = -4b^2/\Delta<0 \\
&= -(1/2)(1/(b(1-2b)))
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
(d/db)(dE/d\Gamma_x) &= (-1/2)(-1)(1-4b)/(b(1-2b))^2 \\
&>0 \text{ for } b>1/4 \\
&<0 \text{ for } 1/4<b<1/2.
\end{aligned} \tag{16}
$$

Due to the increase in the tax rate in the country X, the initial level of the total efforts $\bar{E}^0$ decreases to $\bar{E}^2$ if $b<1/4$ and to $\bar{E}^1$ if $1/4<b<1/2$ holds as shown in Figure (iv).[5]

Finally, it should be mentioned that these results hold only in the neighborhood of the MSY's efforts level.[6] If the change in the technical progress ($dc_i<0$) offsets the tax rate increase ($dt_i>0$), i.e., $dc_i+dt_i<0$, the results will be reversed, even if the tax is imposed.

Footnotes

1 As to the nature of the reguratory measures of fishery resources, see

Anderson (1977). "A Classification of Fishery Management Problems." *Ocean Development and International Law*, Vol. 4, No. 2., Christy & Alexander (1975). "Cooperation in Natural Resources Development: Marine Resources." The paper presented to the 7th Conference on Pacific Trade and Development., pp. 25-27.

2 As to the model developed below, refer to Anderson (1973). "Optimum Economic Yield of an Internationally Utilized Common Property Resources." *Fishery Bulletin*, Vol. 73, No. 1.

See Anderson (1977), *The Economics of Fisheries Management*, The Johns Hopkins University Press.

4 The slope of the line in Figure (iv), i.e., $dE_y/dE_x=-1$, is obtained by differentiating $\pi_x+\pi_y=0$ with respect to E_x and E_y solving for dE_y/dE_x.

5 As to the effect of a landings tax for the one country model, see Flagg (1977). "Alternative Management Plans for Yellowfin Tuna in the Eastern Tropical Pacific." San Diego State University.

6 Flagg (1977). "Optimal Output and Economic Rent of the Eastern Tropical Pacific Tuna Fishery: An Empirical Analysis." *American Journal of Economics and Sociology*, Vol. 36 (Jan.), showed mathematically that as price increases relative to average cost per unit of effort, the difference between maximum sustainable yield and maximum economic yield decreases.